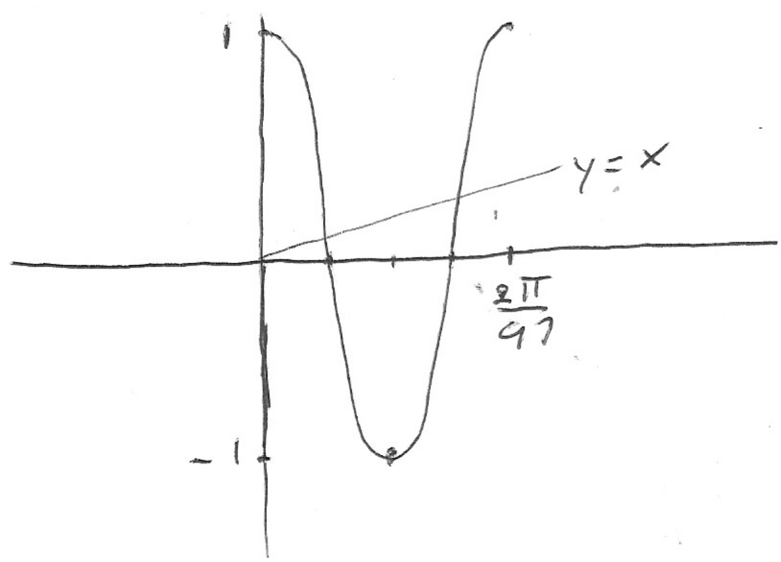
Math 4001 - Fall 2019

A10 - Solutions

15. How many positive numbers x satisfy $\cos(97x) = x$?

The function $y = \cos(97x)$ has period $\frac{2\pi}{97}$ and so it completes one cycle on the interval $[0, \frac{2\pi}{97}]$ as shown:



Since $y = x$ increases from 0 on this interval, there will be two solutions on $[0, \frac{2\pi}{97}]$. The next ~~[illegible]~~ cycle of $\cos \frac{2\pi}{97}$ is on the interval $\left[\frac{2\pi}{97}, \frac{4\pi}{97}\right]$, and there will be two other solutions.

So on, if we let $x_n = \frac{2\pi n}{97}$, then

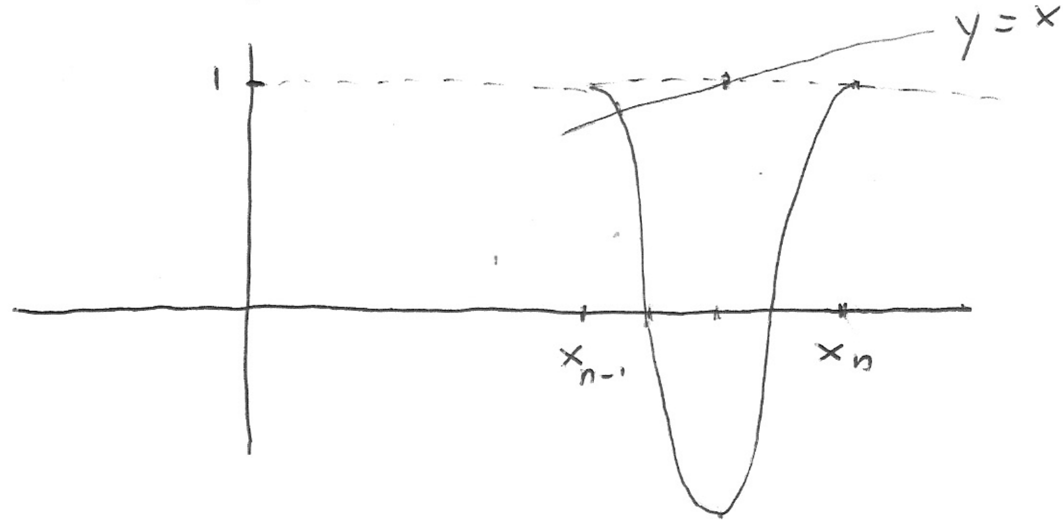
there will be two solutions on

$[x_{n-1}, x_n]$ as long as $x_n < 1$.

But if $x_{n-1} < 1$ and $x_n > 1$, then

there will be only one solution, as

shown in the picture:



So we need to find the smallest

n so that $x_n > 1$. Since

$\frac{97}{2\pi} \simeq 15.43$, we will have

$x_{15} < 1$ and $x_{16} > 1$. So there are

2 solutions in each of the intervals

$[x_0, x_1], [x_1, x_2], \dots, [x_{14}, x_{15}]$ (15 intervals)

and one solution in $[x_{15}, x_{16}]$

for a total of $2(15) + 1 = 31$ solutions